1. Let S be a set of cats. S = {Simon, Traill, Mocha, Lindor, Tigger}

Suppose that no two cats have exactly the same level of intelligence.

We are given this information about the relative intelligence of the cats in S:

Simon is at least as intelligent as Traill.

Traill is at least as intelligent as Lindor.

Traill is at least as intelligent as Tigger.

Mocha is at least as intelligent as Lindor.

Mocha is at least as intelligent as Tigger.

(a) Explain why “is at least as intelligent as” is a partial ordering

(b) Draw a Hasse Diagram to represent the “is at least as intelligent as” relation.

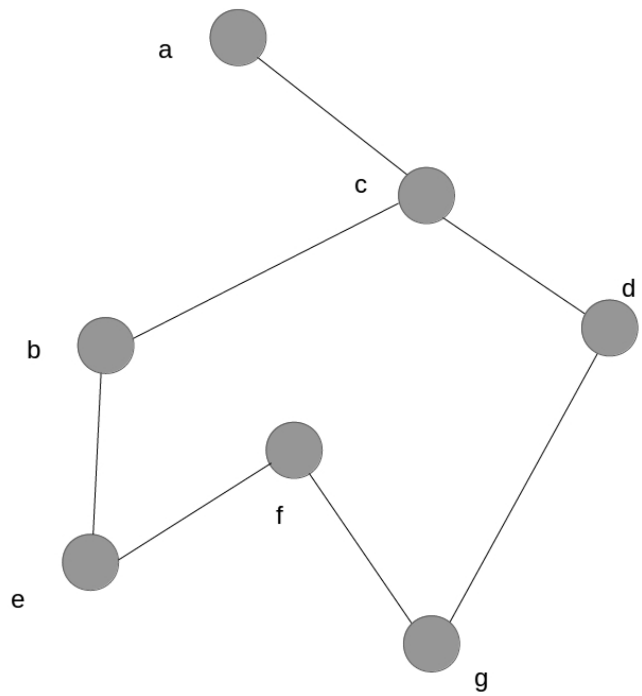
(c) What can we say about the relative intelligence of Traill and Mocha? How about Simon and Mocha?

(d) What is the height of this partial ordering?

(e) What is the width of this partial ordering?

(f) Identify any maximum, minimum, maximal and minimal elements.

2. Here is a Hasse Diagram.



(a) Write out the partial ordering that it represents. (I know, it has more than 20 ordered pairs. Feel free to stop as soon as you are convinced that the Hasse Diagram is a more concise way of representing the partial ordering.)

(b) Identify any maximum, minimum, maximal and minimal elements.

(c) What are the height and width of this partial ordering?

3. Let P be a partial ordering, let C be a chain in P, and let A be an anti-chain in P.

Prove that $|C \cap A| \leq 1$

4. Let $S = \{2, \ldots, 100\}$ and let R be the relation defined by $(a, b) \in R$ if and only if $a = b^i$ where i is a positive integer

For example, $(16, 2) \in R$ and $(100, 10) \in R$

(a) Show that R is a partial ordering of S (ie. show it satisfies the three required properties)

(b) What is the height of this partial ordering? (A little thought here will save you a lot of work!)

(c) If we use $\leq$ to represent this partial ordering, is it true that $81 \leq 3$? Is it true that $25 \leq 36$?